Semi-empirical Dose-Calculation Models in Brachytherapy

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Semi-Empirical Dose-Calculation Models

• Mathematical representation of dose distribution
  – Small number of radionuclide-dependent but source-geometry independent parameters
  – No dose measurements needed to implement for a given source

• History
  – Isotropic point-source kernel and Sievert integral introduced by Sievert, Quimby and Parker in 1920’s
  – Displaced by Monte Carlo- and TLD-based table lookup methods in modern era
  – Accurately represents cesium and iridium dose distributions
‘Model-based’ Dose Calculation

Problem Statement

- Isotropic Point Source
- Air-kerma Strength = $S_K$ \(\mu\text{Gy} \cdot \text{m}^2/\text{h}\)
2004 AAPM Definition of Air-Kerma Strength

\[ S_K = \dot{K}_\delta(d) d^2 \left[ \mu \text{Gy} \cdot \text{m}^{-2} \cdot \text{h}^{-1} = \text{cGy} \cdot \text{cm}^{-2} \cdot \text{h}^{-1} = U \right] \]

\[ \dot{K}_\delta(d) \text{ is air-kerma rate in vacuo due to photons } > \delta \text{ (\(5 \text{ keV}\)) \(d \text{ \(\text{cm}\\text{cm}\)} \text{\text{cm}\text{cm}}} \right) \]

\[ \dot{D}_{\text{air}}(1 \text{ cm}) \cdot (1 \text{ cm})^2 = \dot{X}(1 \text{ cm}) \cdot (1 \text{ cm})^2 \cdot \left( \frac{W}{e} \right) = S_K \cdot (1 - g) \approx S_K \]

\[ \left( \frac{W}{e} \right) = 0.876 \text{ cGy/R} \quad g = \text{Fx of KE emitted as photons} \]
Inverse Square Law

\[ \Phi(r) = \frac{\text{No. incident photons}}{\text{Area irradiated}} = \frac{\text{Activity} \cdot P \cdot \text{Time}}{4\pi r^2} \]

Dose(r) \propto \text{exposure(r)} \propto \Phi(r)

\[ \frac{D(r_1)}{D(r_2)} = \frac{\Phi(r_1)}{\Phi(r_2)} = \left( \frac{r_2}{r_1} \right)^2 \]

\[ \left( \frac{\dot{D}_{\text{med}}}{\dot{D}_{\text{air}}} \right)_{\text{free space}} = \left( \frac{\mu_{\text{en}}/\rho}{\mu_{\text{en}}/\rho} \right)_{\text{med}} / \left( \mu_{\text{en}}/\rho \right)_{\text{air}} = \left( \mu_{\text{en}}/\rho \right)_{\text{air}}^{\text{med}} \]
Tissue Attenuation Factor, $T(r)$

- Describes competition between primary photon and attenuation and scatter buildup
- $T(r) = 1 \pm 0.05$ for $r < 5$ cm when $E > 200$ keV
- Often derived from 1D transport calculations

$$T(r) = \frac{\dot{D}_{\text{wat}}(r) \text{ in Water}}{\dot{D}_{\text{wat}}(r) \text{ in Vacuum}}$$
Isotropic Point-Source Kernel

- Assuming air-kerma strength calibration

\[
\dot{D}_{\text{med}}(r) = \frac{S_K \cdot (\mu_{\text{en}} / \rho)_{\text{air}}^{\text{med}}}{r^2} \cdot T(r)
\]

- Assuming contained activity calibration

Exposure-rate constant: \( (\Gamma_\delta)_x \frac{X_\delta(r) \cdot r^2}{A} \)

\[
\dot{D}_{\text{med}}(r) = A \cdot \frac{(\Gamma_\delta)_x (W / e) \cdot (\mu_{\text{en}} / \rho)_{\text{air}}^{\text{med}}}{r^2} \cdot T(r) = A \cdot \frac{(\Gamma_\delta)_x \cdot f_{\text{med}}}{r^2} \cdot T(r)
\]
TG-43 and Classical Point-kernel

\[ \Lambda \equiv \frac{\dot{D}(1 \text{ cm}, \pi/2)}{S_K} \approx \frac{(\mu_{en}/\rho)_{\text{wat}} \cdot T(1 \text{ cm}) \cdot G_L(1 \text{ cm}, \pi/2)}{D(1 \text{ cm}, \pi/2) \cdot G_L(r, \pi/2)} \approx \frac{T(r)}{T(1 \text{ cm})} \]

- Generally, CPK and modern Monte Carlo-TLD dosimetry agreement within 2%-3% along transverse-axes of $^{192}$Ir and $^{137}$Cs sources
## TLD-MC vs. Classical For $^{192}$Ir

### Anctil 1998

![Graph showing radial dose function vs. distance for various studies.]

### Valicenti 1995

<table>
<thead>
<tr>
<th>Author</th>
<th>Method</th>
<th>Measurement medium</th>
<th>$\Lambda$ (cGy h$^{-1}$ U$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Williamson</td>
<td>Monte Carlo</td>
<td>30 cm liquid water sphere</td>
<td>1.128 ± 0.5%</td>
</tr>
<tr>
<td>Present study</td>
<td>TLD dosimetry</td>
<td>Solid-Water phantom</td>
<td>1.122 ± 4.0%</td>
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<tr>
<td>Low dose rate interstitial seeds (Steel Clad)</td>
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<td></td>
<td></td>
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<tr>
<td>Williamson</td>
<td>Monte Carlo</td>
<td>Liquid water: unbounded</td>
<td>1.110 ± 0.2%</td>
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<tr>
<td></td>
<td></td>
<td>Solid Water: unbounded</td>
<td>1.121 ± 0.3%</td>
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<tr>
<td>Nath</td>
<td>TLD dosimetry</td>
<td>Solid Water: Nath phantom</td>
<td>1.119 ± 0.2%</td>
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<tr>
<td>Weaver</td>
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<td>Solid Water</td>
<td>1.12 ± 2.7%</td>
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<td>Chiu-Tsao</td>
<td>TLD dosimetry</td>
<td>Liquid Water: unbounded</td>
<td>1.111 ± 1.5%</td>
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<tr>
<td>Meisberger</td>
<td>Scintillation probe, transport calculation</td>
<td></td>
<td>1.118</td>
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</tbody>
</table>

Table II: Dose rate constant for Ir-192 sources.
Δ\( \hat{D}(x,y) = S_K \cdot \frac{\Delta L}{L} \cdot \frac{\left( \mu_{en} / \rho \right)_{\text{wat}}}{(x / \cos \theta)^2} \cdot e^{\mu \cdot t} \cdot T(x / \cos \theta)e^{-\mu \cdot t / \cos \theta} \)
Sievert Filtered Line-Source Integral

\[ \dot{D}(x,y) = S_K \cdot \frac{(\mu_{en}/\rho)^{wat}}{L \cdot x} \cdot e^{\mu \cdot t} \cdot \int_{\theta_1}^{\theta_2} e^{-\mu \cdot t \cdot \sec \theta} \cdot T(x \cdot \sec \theta) \cdot d\theta \]

- \( \mu = \) effective filtration coefficient (0.021 to 0.039 mm\(^{-1}\) for Cs-137 and steel)
  - Best results: treat \( \mu \) as fitting parameter, not assume \( \mu = \mu_{en} \)
- \( L = \) active length, \( t = \) filter thickness
- Sievert model works well for Cs-137, but \( \Rightarrow >10\% \) errors for lower energy sources off transverse axis
Modern 1D Pathlength Algorithms

- Integrate CDK over 3D source geometry

Diagram showing actual geometry compared to cylindrical core and steel line source.
Modern 3D Pathlength Model with Best Fit

- CDCS-J 1992 Source
- 3D geometric Model
- Best Fit Filtration Coef.
- RMS error: 0.8%
- -0.9% to 4.1% range
‘Conventional’ Sievert Model

- CDCS-J 1992 Source
- Ceramic Line Source Model
- Filtration \( \mu_{en} \)
- RMS error: 3.0%
- -5.4% to 10.1% range
1D Pathlength Model vs. Monte Carlo

3M 6500 $^{137}$Cs Tube

Model 3M 6500 Source: 2 cm Polar profile

Dose Rate/Air-Kerma Strength (cGy/h per unit U) vs. Angle (degrees)

- Monte Carlo
- Best fit
- $\mu_{en}$
- Best Fit
- $\mu_{en}$

3D Geometry

Ceramic Line Source
1D Pathlength Model: HDR $^{192}$Ir

- %RMS error = 6.9%
Scatter Separation Model: HDR $^{192}$Ir
Williamson IJROBP 1996

- %RMS error = 2.9%
- Max error = 8.9%

$\dot{D}(r, \theta) = \text{primary dose at } (r, \theta) + \text{transverse axis scatter dose at } (r, \pi/2)$

$\dot{D}(r, \theta) = \dot{D}_{\text{pri}}(r, \theta) + \dot{D}_{\text{pri}}(r, \pi/2) \cdot \text{SPR}(r)$
Sievert Model: $^{125}$I Seed

Best-fit constant filtration coefficients

$\mu_{Ti} = 1.825 \, \text{mm}^{-1}$

$\mu_{\text{Resin}} = 0.00 \, \text{mm}^{-1}$
Why is 6711 I-125 $\Lambda$ lower?

- Classical

$$\Lambda = \left( \frac{\mu_{en}}{\rho} \right)_{\text{air}}^{\text{med}} \cdot T(r_0)$$

$$= 1.012 \cdot 1.012 = 1.024$$

- 1995 TG-43 $\Lambda_{85N,95D} = 0.88$ 15% lower!
  - 1985 NIST $S_K$ Standard contaminated by 4.5 keV Ti characteristic x-rays (10% effect)
  - Conventional $T(r)$ neglects 22 keV Ag characteristic x-rays (5% effect)
1D pathlength Model
Shielded Fletcher-Suit Colpostat

- Weeks 1998: Best-fit $\mu$ for tungsten
  - Solid Line: MCNP Monte Carlo
  - Broken Line: 1D pathlength model
Recent Model-based Algorithms

• Scatter separation
  – Williamson 1996: improves accuracy of $^{192}$Ir anisotropy functions

• Superposition/Convolution
  – Williamson 1991; Carlsson 2000
  – Accurate but slow

• Boltzmann transport equation solutions
  – Discrete ordinates: Daskalov/Williamson
  – Fast Monte Carlo: Le 2005, Chibani 2005
  – Integral transport: Zhou 2004
Current single-source dose superposition algorithm: patients are composed of homogeneous liquid water

- Cs-137 & Ir-192 sources: 15%-50% applicator, shielding, bounded scatter-volume effects
- Pd-103/I-125 interstitial sources: 5%-50% interseed attenuation and tissue inhomogeneity effects
Tissue Heterogeneity Effects
78 Model 6711 $^{125}$I seeds
Single-Energy CT Material Analysis (CTcreate)
## Efficiency Performance

LINUX with single Xeon 3.0 GHz processor

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Code</th>
<th>% $\sigma$</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>77×64×81 mm$^3$</td>
<td>PTRAN_744</td>
<td>1.5</td>
<td>412</td>
</tr>
<tr>
<td>1×1×1 mm$^3$</td>
<td>PTAN_CT</td>
<td>1.5</td>
<td>71</td>
</tr>
<tr>
<td>42×35×45 mm$^3$</td>
<td>PTRAN_744</td>
<td>1.5</td>
<td>76</td>
</tr>
<tr>
<td>2×2×2 mm$^3$</td>
<td>PTAN_CT</td>
<td>1.5</td>
<td>15</td>
</tr>
<tr>
<td>30×30×30 mm$^3$</td>
<td>PTRAN_744</td>
<td>1.5</td>
<td>35</td>
</tr>
<tr>
<td>2×2×2 mm$^3$</td>
<td>PTAN_CT</td>
<td>1.5</td>
<td>6</td>
</tr>
<tr>
<td>Prostate</td>
<td>PTAN_CT</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

PTRAN_CT vs. PTRAN: 5 times more efficient
PTRAN_CT Features

- **Precomputed “Phase space” source**
  - Primary photons tracked to surface of source/applicator with unit weight saved. Used to initialize photon histories during patient simulations

- **Ray tracing**
  - Fast parametric ray tracing through voxel grid (Siddon 1985)
  - Preprocessing: identifies voxels that intersect seeds and applicators
  - Analytic ray tracing limited to applicators contiguous to intersected voxels
  - Fast algorithm for merging analytic and voxel path lengths
Phase Space Concept

Precomputed single-source histories

Effective primary photons for patient simulation
PTRAN_CT Ray Tracing

Analytic Ray Tracing

4 regions and 8 surfaces/ 6711 seed x 78 seeds

+ 

Voxel grid Tracing

Voxel grid Ray Tracing
77×64×81 grid = 400K cells

= 

integrated Ray Tracing
Conclusions

• Isotropic point-kernel and 1D pathlength models are accurate TG 43 alternatives
  – Interstitial seeds $\geq$ $^{192}$Ir photon energies
  – $^{137}$Cs intracavitary tubes and shielded colpostats including 2D anisotropy
  – $^{192}$Ir anisotropy functions when 1D pathlength + scatter separation is used

• Semi-empirical model limitations
  – Require careful benchmarking against Monte Carlo to validate for a class of problems
  – Not validated for lower-energy sources
  – More sophisticated models may not be competitive with Monte Carlo dose-calculation engines
The Future: Monte Carlo-based Dose-Calculation

- Subminute Monte Carlo dose calculation for full clinical implant geometries possible
- Implementation issues
  - More complex applicator/seed localization
  - RTP input will be source/applicator geometry or phase space, not single-source dose distribution
- Prostate/low energy seed issues
  - How to measure tissue composition? dual-energy CT?
  - CT artifact mitigation
    » Urethral contrast biases MC
    » Streaking artifacts from seeds mimic bone
Tandem & Colpostat Localization

- Weeks CT-compatible applicators with aluminum bodies, afterloading tungsten shields, & Cs-137 tubes
- Find applicator pose that maximizes coincidence of Monte Carlo applicator geometry with CT contours of applicators