# IMRT Optimization Based on Physical Criteria

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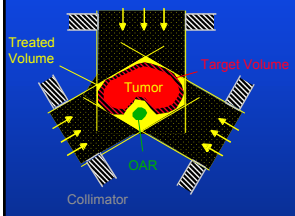
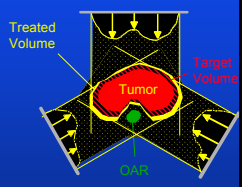
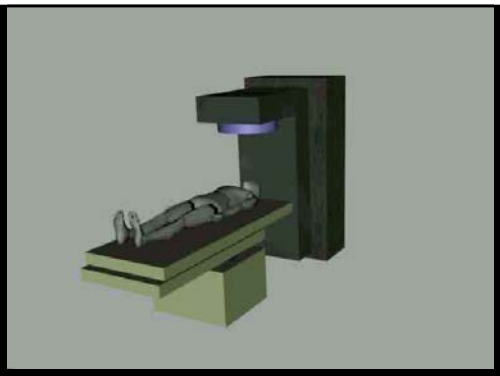
## Motivation

3D conformal Rx




Intensity Modulation



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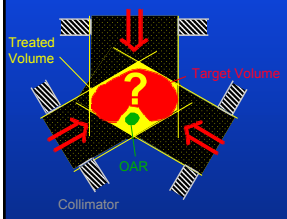
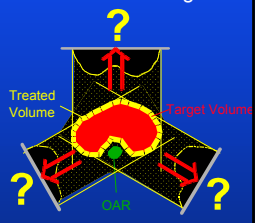
Forward Planning




Inverse Planning




## Outline

1. Physical optimization criteria
2. The objective function
3. Variables to be optimized
4. Optimization algorithm: An example

## 1. Physical optimization criteria

1.1 Clinically relevant criteria
- 1.1.1 Deviation from prescribed dose
- 1.1.2 Target dose homogeneity issues
- 1.1.3 Min/Max dose
- 1.1.4 Mean dose
- 1.1.5 DVH criteria
- 1.1.6 EUD

1.2 Physical and technical criteria
- 1.2.1 Non-negativity of intensities
- 1.2.2 Deliverability, smoothness of IM maps

## 1.1.1 Deviation from prescribed dose

### Least squares deviation

$$F(\vec{b}) = \sum_i \left(d_i(\vec{b}) - d_i^p\right)^2$$

ray intensities, beam angles, . . . | dose at voxel $i$ | prescribed dose

Clinically this is NOT very useful!

## 1.1.1 Deviation from prescribed dose

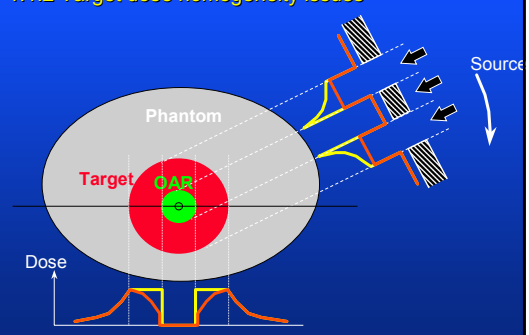
$$F_T(\vec{b}) = \sum_{i=1}^{N_T} \left( u\left[D_{\min} - d_i(\vec{b})\right]_+^2 + w\left[d_i(\vec{b}) - D_{\max}\right]_+^2 \right)$$

underdose

overdose

"Positivity operator": $[a]_+ = \begin{cases} a & \text{for } a \geq 0 \\ 0 & \text{otherwise} \end{cases}$

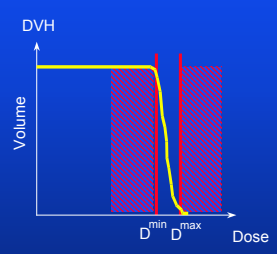
## 1.1.2 Target dose homogeneity issues



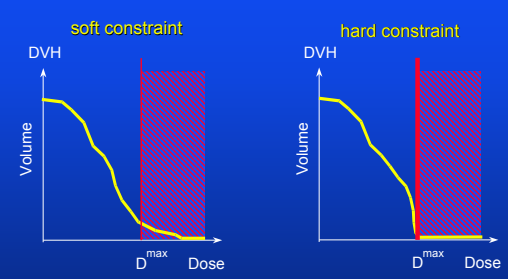

## 1.1.2 Target dose homogeneity issues

- IMRT does not necessarily lead to bigger target dose inhomogeneity
- For complex cases, IMRT is the only way to make target dose uniform
- In practice, IMRT target dose distributions are often less homogeneous than 3DCRT
  - Margins
  - Number of beams
  - Tradeoff in critical structures
  - Objective function

## 1.1.3 Min/Max doses



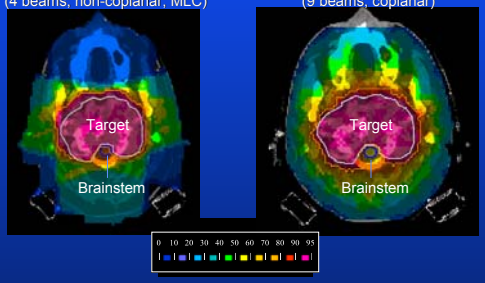

## 1.1.3 Min/Max doses



## 1.1.3 Min/Max doses – Example: Clivus Chordoma

Non-intensity-modulated
(4 beams, non-coplanar, MLC)

Intensity-modulated
(9 beams, coplanar)



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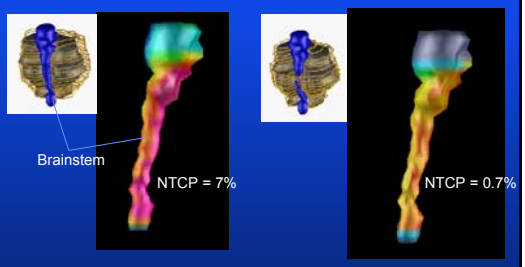
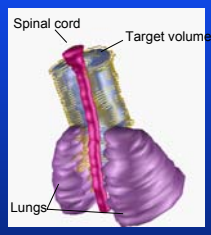
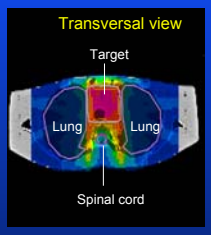





## 1.1.3 Min/Max doses – Example: Thyroid

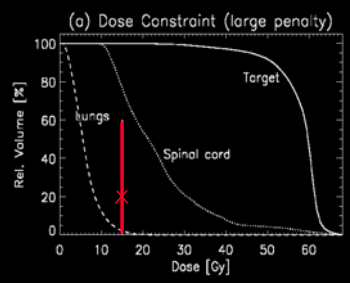
Technique: 9 beams, coplanar, intensity-modulated






## 1.1.3 Min/Max doses – Example: Thyroid



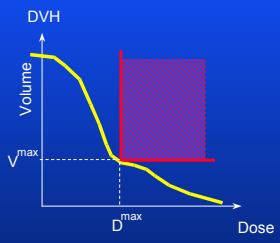



## 1.1.4 Mean dose

- The mean dose is a good descriptor of the dose effect in parallel critical structures such as lung
  - S.L. Kwa et al.: "Radiation pneumonitis as a function of the mean lung dose", IJROBP 42(1):1-9, 1998



## 1.1.5 Dose-volume criteria



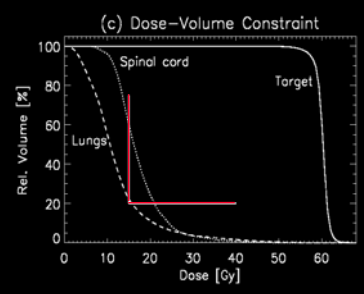
## 1.1.5 Dose-volume criteria

"No more than 1/3 of the lung should get more than 15 Gy."

$$\mathrm{DVH}_k(D_k^{\max}) \le V_k^{\max}$$



## 1.1.5 Dose-volume criteria – Example: Thyroid






## 1.1 Physical criteria in clinical protocols: RTOG-022 oropharyngeal cancer

Target dose prescription (homogeneity):

- No more than 1% of planning target volume (PTV) can receive less than 93% of prescription
- No more than 20% of the PTV can receive greater than 110% of prescription
- No more than 1% or 1cc of tissue outside of PTVs will receive greater than 110% of prescription dose to primary PTV

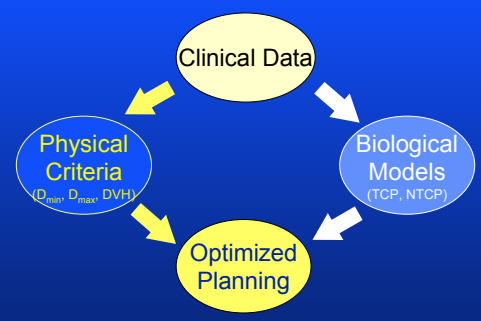
Critical Structures dose limits:

- Brainstem: 54 Gy
- Cord (+ 5mm): 45 Gy
- Mandible: 70Gy

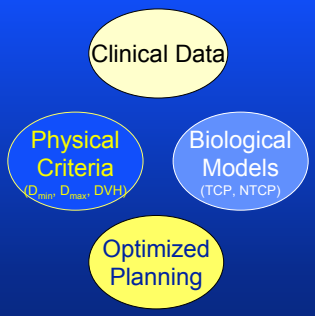
Parotids (3 options):

- Mean dose to either parotid < 26 Gy or
- At least 50% of either parotid gland < 30 Gy or
- At least 20 cc of the combined parotid volume will receive < 20Gy

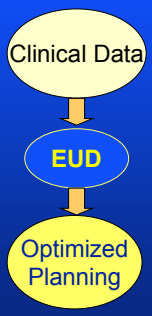








## 1.1.5 EUD

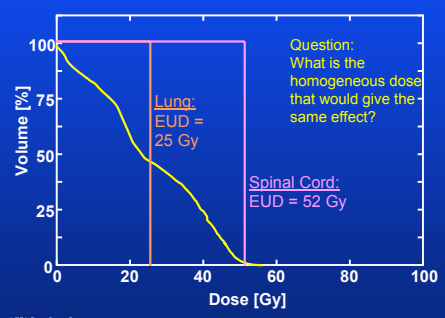
- The equivalent uniform dose (EUD) is the uniform dose the causes the same effect as the given non-uniform dose

## 1.1.5 EUD

Power-law relationship for tolerance dose (TD):

$$\mathrm{TD}(v) = \frac{\mathrm{TD}(1)}{v^n}$$

## 1.1.5 EUD




## 1.1.5 EUD

Brahme et al. ($D_{eff}$), Acta Radiol. Oncol. 23(5), 379-391, 1984
Mohan et al., Med. Phys. 19(4), 933-944, 1992
Kwa et al., Radiother. Oncol. 48(1), 61-69, 1998
Niemierko, Med. Phys. 26(6), 1100, 1999

$$\mathrm{EUD} = \left( \sum_i v_i \cdot D_i^a \right)^{1/a}$$

"a-norm"
generalized mean

Examples:

$$a = 1: \quad \mathrm{EUD} = \overline{D}$$

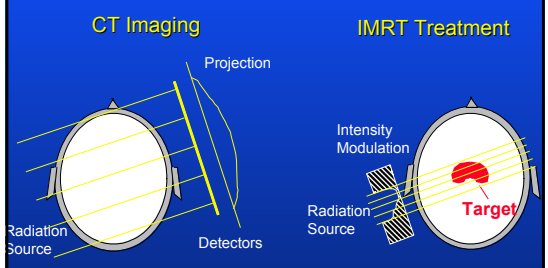
$$a = \infty: \mathrm{EUD} = D_{\max}$$

## 1.1.5 EUD

- Use generalized Equivalent Uniform Dose (EUD) with negative "a" also for target volumes
  - A. Niemierko "A generalized concept of equivalent uniform dose (EUD)" Med. Phys. 26:1100, 1999

## 1. Physical optimization criteria

## 1.2.1 Non-negativity of intensities – The CT analogy




## 1.2.2 Deliverability – smoothness of intensity maps

Original

After smoothing

Dose distributions are almost identical

## 1.2.2 Deliverability – smoothness of intensity maps

- Include smoothness term in objective function
  - M. Alber, F. Nusslin: "Intensity Modulated Photon Beams Subject to a Minimal Surface Constraint", Phys. Med. Biol. 45, N49-N52, 2000
- Apply smoothing filter during optimization
  - S. Webb: "Inverse Planning with Constraints to Generate smoothed intensity-modulated beams", Phys. Med. Biol. 43, 2785-2794, 1998
  - A. Kessen, K.-H. Grosser, T. Bortfeld: "Simplification of IMRT intensity maps by means of 1-D and 2-D median filtering...", Proceedings of ICCR 2000

## 2. The objective function

2.1 Combination of criteria using weights

2.2 Constraints, feasibility search

2.3 Mono-criteria vs. multi-criteria

2.4 A new multi-criteria optimization concept

## 2.1 Combination of criteria using weights - Target

underdose

$$F_T(\vec{b}) = \sum_{i=1}^{N_T} \left( u\left[D_{\min} - d_i(\vec{b})\right]_+^2 + w\left[d_i(\vec{b}) - D_{\max}\right]_+^2 \right)$$

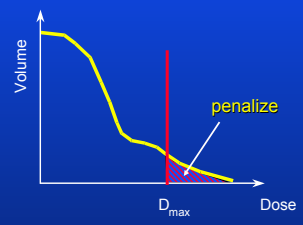
overdose

"Positivity operator": $[x]_+ = \begin{cases} x & \text{for} \quad x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

- This is not the only or the "best" measure of deviations from prescription or tolerance.
- For example, instead of the quadratic deviation, higher powers (4, 6, …) have been suggested (see the work from Ann Arbor, Fraass et al 2002)

## 2.1 Combination of criteria using weights - OAR

Max dose




## 2.1 Combination of criteria using weights - OAR

"Penalty" function:

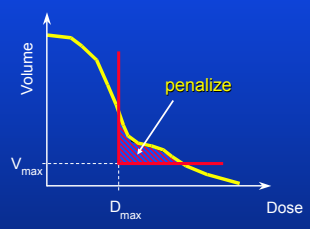
$$F_{R,k}(\vec{b}) = w_k \sum_{i=1}^{N_{R,k}} \left[ d_{i,k}(\vec{b}) - D_{\max,k} \right]_+^2$$

penalty — dose at voxel *i* in OAR *k* — tolerance dose

"Positivity operator": $[x]_+ = \begin{cases} x & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

## 2.1 Combination of criteria using weights - OAR

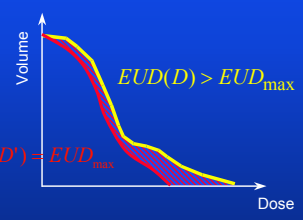
DVH criteria




Helios, BrainLab, KonRad, …

## 2.1 Combination of criteria using weights - OAR

EUD criteria

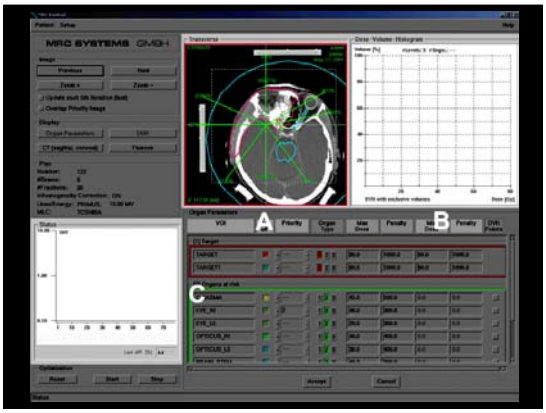



C. Thieke et al., Med. Phys, in print

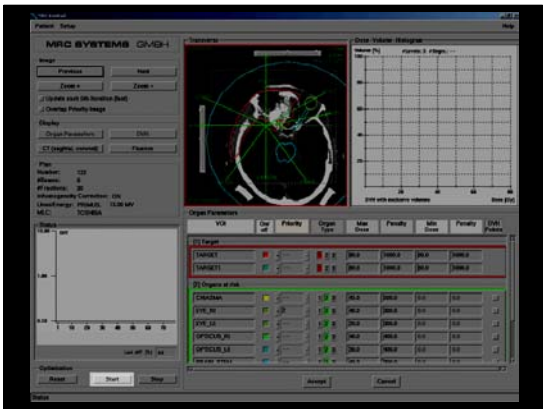
## 2.1 Combination of criteria using weights

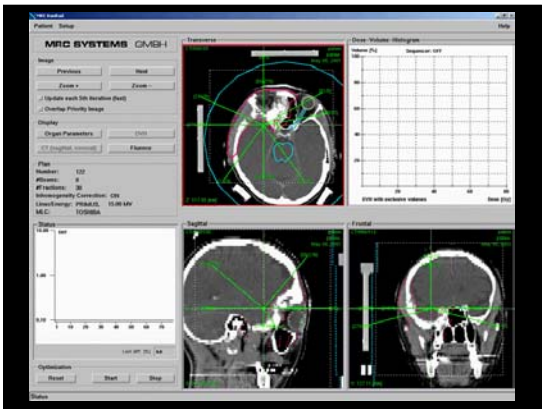
Optimize (minimize)

$$F = w_{\text{Target}} F_{\text{Target}} + w_{\text{Risk1}} F_{\text{Risk1}} + w_{\text{Risk2}} F_{\text{Risk2}} + \ldots$$








## 2. The objective function





## 2.2 Constraints, feasibility search

- Feasibility search:
  Find a plan the fulfills all constraints – no optimization
- See previous presentation by Yair Censor

## 2. The objective function

2.1 Combination of criteria using weights
2.2 Constraints, feasibility search
2.3 Mono-criteria vs. multi-criteria
2.4 A new multi-criteria optimization concept

## Problems with weights (penalties) and constraints

Optimization using weights
- Trial and error to determine weights
- "Sensitivity" of the solution

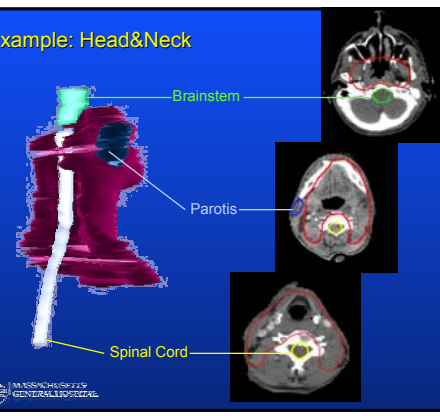
Constrained optimization
- Non-efficient solutions

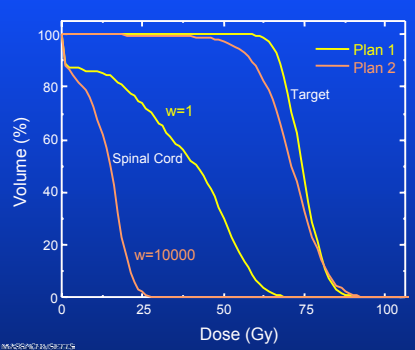
## Problems with weights (penalties) and constraints

$$F = w_{\text{Target}} F_{\text{Target}} + w_{\text{Risk1}} F_{\text{Risk1}} + w_{\text{Risk2}} F_{\text{Risk2}} + \ldots$$

$F$ is a single number!

## Example: Head&Neck

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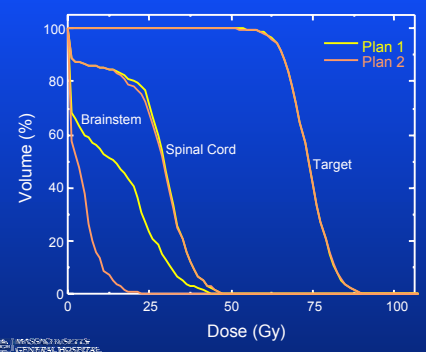
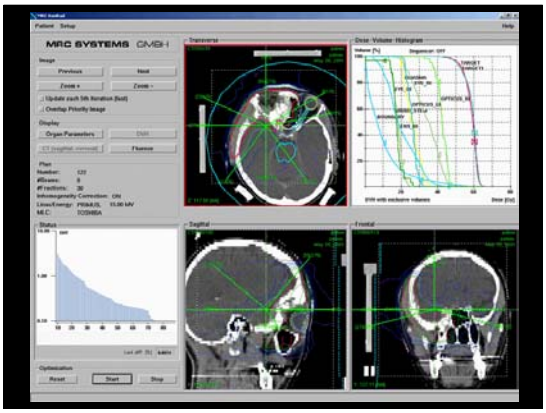
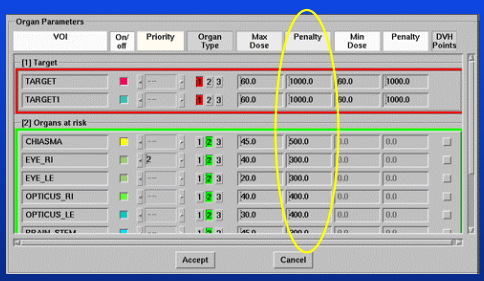





## Change "penalties" or "weight factors"






## 2.3 Mono-criteria vs. **multi-criteria**

- Radiotherapy planning (and IMRT planning) is a multi-criterial problem
- It always involves tradeoffs, one can never optimize all criteria at the same time
- A treatment plan cannot be scored with a single "grade"

➢ Optimization for multi-criteria problems "efficiency", Pareto-optimality



## Pareto optimization

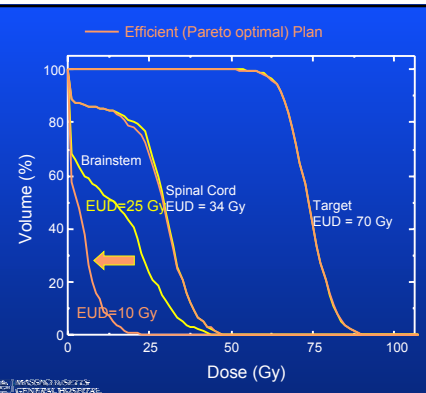
Vilfredo Pareto, 1848-1923

Italian-Swiss socio-economist

Pareto-optimality, "efficient":
You cannot make anybody better off without making someone else worse off.

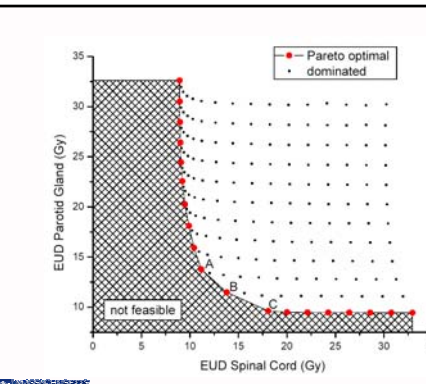
Equilibrium

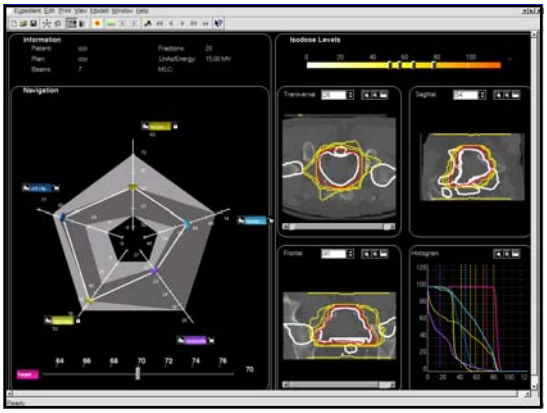
- A Pareto-optimal or "efficient" treatment plan is a plan in which we cannot improve one aspect (e.g., reduce the dose in one OAR) without compromising at least one other aspect (e.g., reduce the target dose)




Towards multi-criteria optimization

- Head & Neck example
- Hard constraint: target EUD = 60 Gy, homogeneity criteria
- Vary EUD constraints for Parotis and spinal cord from 0 Gy to 30 Gy in steps of 2 Gy
- 17 hours for 256 plans



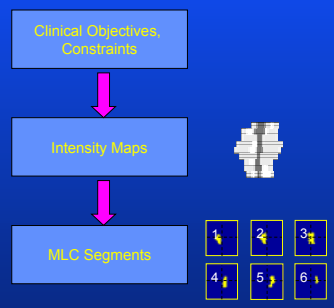
Outline

1. Physical optimization criteria
2. The objective function
3. Variables to be optimized
4. Optimization algorithm: An example

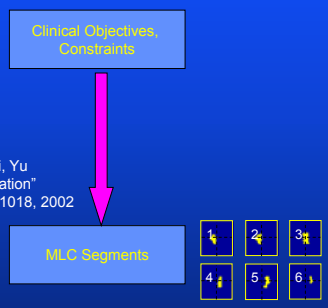
## 3. Variables to be optimized

- Intensity profiles
- MLC shapes, segment weights
- Number of beams
- Beam angles (gantry angle, table angle)
- Number of intensity levels
- Energy (mainly in charged particle therapy)
- Type of radiation (photons, electrons, ...)
- …

## Current IMRT approach (Divide and Conquer)

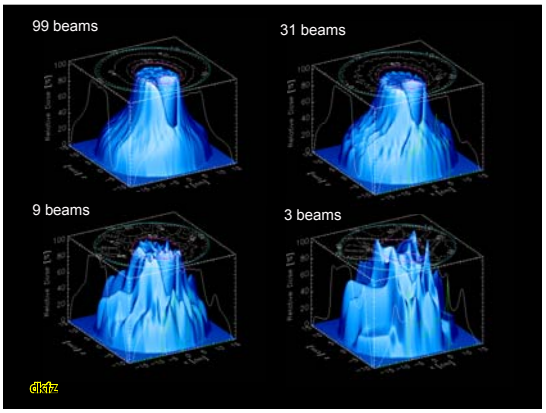



## Challenge: Aperture IMRT



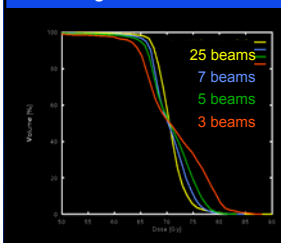

J. Galvin, W. De Neve

Shepard, Earl, Li, Naqvi, Yu
"Direct aperture optimization"
Med. Phys. 29(6):1007-1018, 2002

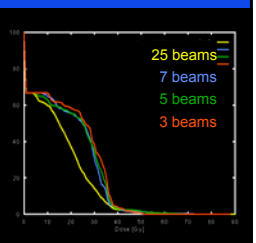



## Example: Clivus chordoma

Target



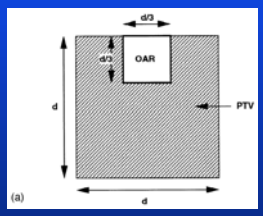

Brainstem



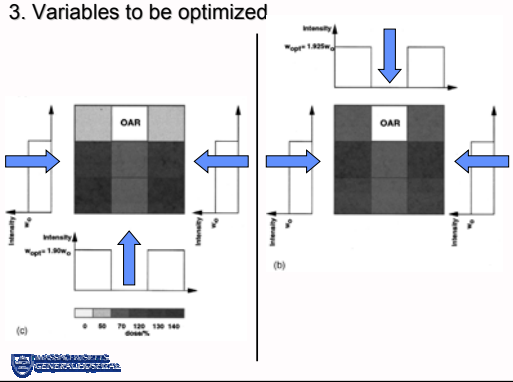

## 3. Variables to be optimized

Beam angles

J. Stein et al.: Number and orientation of beams in IMRT
Med. Phys. 24(2):149-160, 1997



## 3. Variables to be optimized




## Beam angles, number of beams

- If many beams ($\geq 9$) are used, the selection of beam angles is uncritical:
  - evenly spaced coplanar beams can be used
- For smaller numbers of beams,
  - Careful selection (optimization) of beam angles can lead to improvements
- The number is case-dependent
  - Pugachev et al., "Role of beam orientation optimization in IMRT" IJROBP 50(2): 551-60, 2001

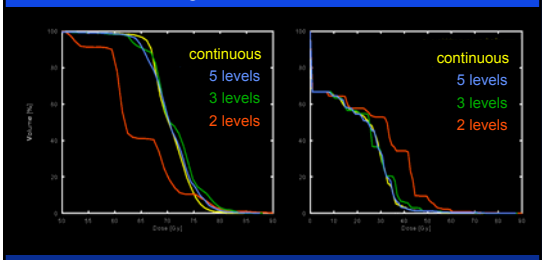
## Number of intensity levels

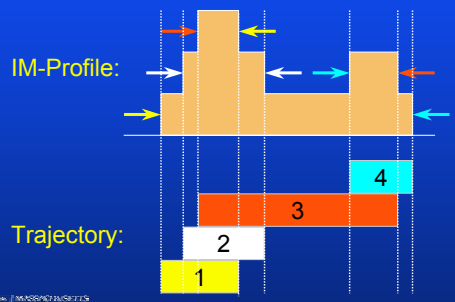
Keller-Reichenbecher et al.
"Intensity modulation with the "step and shoot" technique"
Int. J. Radiat. Oncol. Biol. Phys. 45:1315-1324, 1999


Beam 1 of 7 (0°)

## Number of intensity levels, Example: Clivus chordoma

7 Beams, Target 7 Beams, Brainstem




## Energy

- Energy plays a surprisingly small role in IMRT. 6MV is a good choice for many cases.
  - E.S. Sternick, et al. "IMRT: What Photon Energy is best?", Proc. of ICCR 12, Med. Phys. Publishing. pp 418-419, 1997

## Outline

1. Physical optimization criteria
2. The objective function
3. Variables to be optimized
4. Optimization algorithm: An example



## 4. Optimization algorithm: An example

$$F(\vec{b}) = \sum_{i=1}^{N} w_i \left[ d_i(\vec{b}) - P_i \right]^2$$

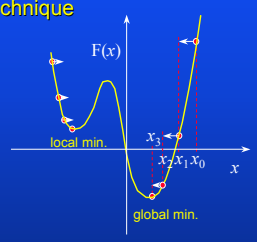
$$d_i(\vec{b}) = \sum_j D_{ij} b_j$$

$D_{ij}$ : dose contribution of pencil beam *j* to voxel *i*



## 4. Optimization algorithm: An example

Gradient technique




$$\vec{x}_{t+1} = \vec{x}_t - \alpha \cdot \mathrm{grad}\{F(\vec{x})\}\Big|_{\vec{x}_t}$$

1D: $x_{t+1} = x_t - \alpha \cdot \dfrac{dF}{dx}$



## 4. Optimization algorithm: An example

Gradient:

$$\frac{\partial F}{\partial b_j} = 2\sum_{i=1}^{N} w_i (d_i - P_i) D_{ij}$$

2nd derivative:

$$\frac{\partial^2 F}{\partial b_j^2} = 2\sum_{i=1}^{N} w_i D_{ij}^2$$



## 4. Optimization algorithm: An example

<u>Algorithm (Newton-like):</u>

$$b_j^{t+1} = \left[ b_j^t - \alpha \frac{\frac{\partial F}{\partial b_j}}{\frac{\partial^2 F}{\partial b_j^2}} \right]_+ \qquad \alpha = 1/N_B$$

$$= \left[ b_j^t + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i^t) D_{ij}}{\sum_{i=1}^{N} w_i D_{ij}^2} \right]_+$$



## 4. Optimization algorithm: An example

A <u>very</u> simple example:

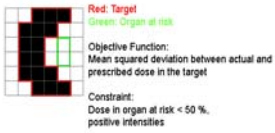
Dose distribution:

$$D_{ij} = \begin{cases} 1 & \text{if pencil beam (ray) } j \text{ hits voxel } i \\ 0 & \text{otherwise} \end{cases}$$

Initial values:

$$b_j^0 = 0$$

$$d_i^0 = 0$$

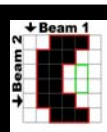


Red: Target
Green: Organ at risk

Objective Function:
Mean squared deviation between actual and prescribed dose in the target

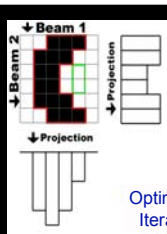
Constraint:
Dose in organ at risk < 50 %,
positive intensities

Optimization
Iteration 1




Optimization
Iteration 1

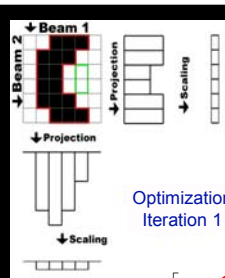
$$b_j^1 = \left[ b_j^0 + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i^0) D_{ij}}{\sum_{i=1}^{N} w_i D_{ij}^2} \right]_+$$




Optimization
Iteration 1

$$b_j^1 = \left[ b_j^0 + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i^0) D_{ij}}{\sum_{i=1}^{N} w_i D_{ij}^2} \right]_+$$

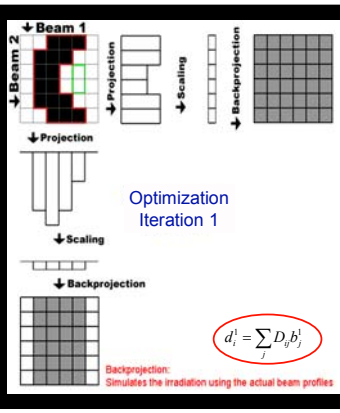
Projection: Summation of the dose values along the beam direction




Optimization
Iteration 1

$$b_j^1 = \left[ b_j^0 + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i^0) D_{ij}}{\sum_{i=1}^{N} w_i D_{ij}^2} \right]_+$$

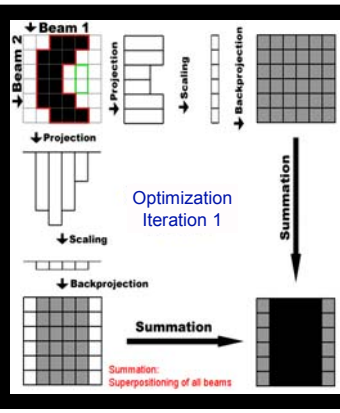
Scaling: Division by number of contributing dose elements
and by number of beams




Optimization
Iteration 1

$$d_i^1 = \sum_j D_{ij} b_j^1$$

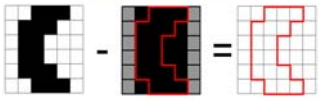
Backprojection:
Simulates the irradiation using the actual beam profiles




Optimization
Iteration 1

Summation:
Superpositioning of all beams

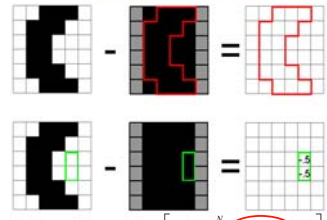
## Optimization Iteration 2



$$b_j^2 = \left[ b_j^1 + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i^1) D_{ij}}{\sum_{i=1}^{N} w_i D_{ij}^2} \right]_+$$

Comparing prescribed and realized dose in the target volume.
In this case there is no difference, therefore no correction necessary.

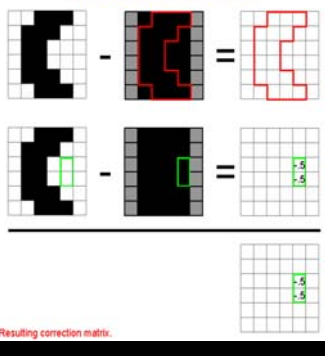
## Optimization Iteration 2



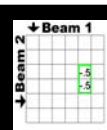

$$b_j^2 = \left[ b_j^1 + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i^1) D_{ij}}{\sum_{i=1}^{N} w_i D_{ij}^2} \right]_+$$

Same for the organ at risk: Here the difference is -0.5.

## Optimization Iteration 2

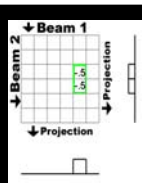



Resulting correction matrix.



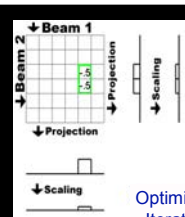

## Optimization Iteration 2

$$b_j^2 = \left[ b_j^1 + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i^1) D_{ij}}{\sum_{i=1}^{N} w_i D_{ij}^2} \right]_+$$



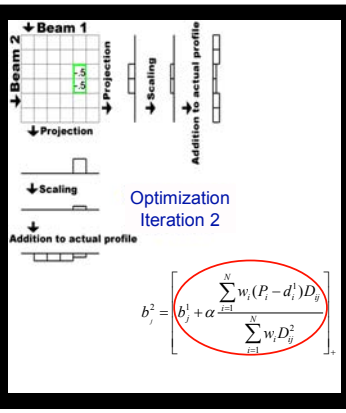

## Optimization Iteration 2

$$b_j^2 = \left[ b_j^1 + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i^1) D_{ij}}{\sum_{i=1}^{N} w_i D_{ij}^2} \right]_+$$

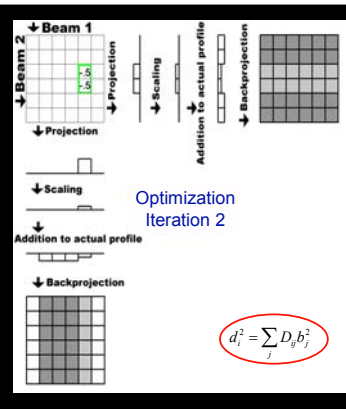



## Optimization Iteration 2

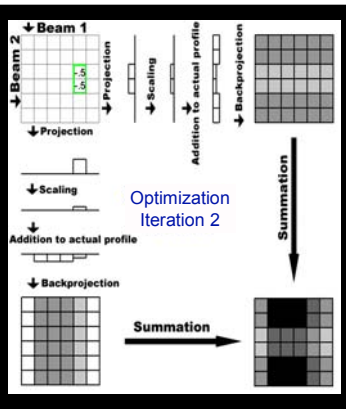
$$b_j^2 = \left[ b_j^1 + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i^1) D_{ij}}{\sum_{i=1}^{N} w_i D_{ij}^2} \right]_+$$

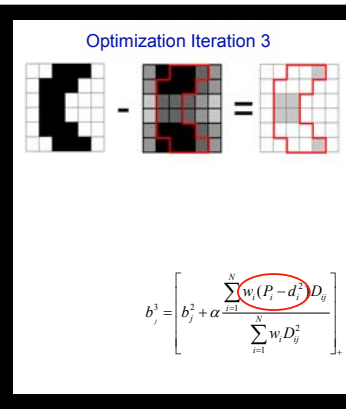



$$b_j^2 = \left[ b_j^1 + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i^1) D_{ij}}{\sum_{i=1}^{N} w_i D_{ij}^2} \right]_+$$

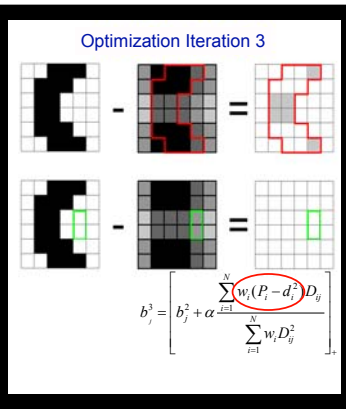



$$d_i^2 = \sum_j D_{ij} b_j^2$$

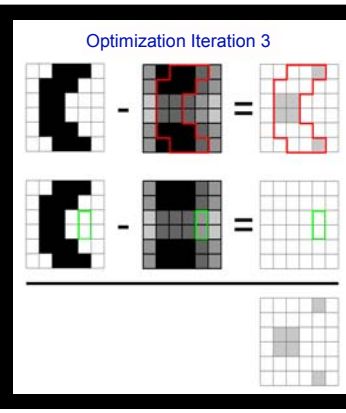



$$b_j^3 = \left[ b_j^2 + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i^2) D_{ij}}{\sum_{i=1}^{N} w_i D_{ij}^2} \right]_+$$




$$b_j^3 = \left[ b_j^2 + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i^2) D_{ij}}{\sum_{i=1}^{N} w_i D_{ij}^2} \right]_+$$



Optimization
Iteration 3

$$b_j^3 = \left[ b_j^2 + \alpha \frac{\sum_{i=1}^{N} w_i (P_i - d_i^2) D_{ij}}{\sum_{i=1}^{N} w_i D_{ij}^2} \right]_+$$

Repeat the steps of evaluation and optimization until an acceptable treatment plan is found.

## 4. Optimization algorithms

- Projecting back and forth between dose distribution and intensity maps



## Summary

1. Physical optimization criteria
2. Objective functions
3. Variables to be optimized
4. Optimization algorithm: An example