3D Filtered Backprojection
Fundamentals, Practicalities, and Applications

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Acknowledgments

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Evolution and Proliferation of CT

Sir Godfrey Hounsfield
Nobel Prize, 1979
Evolution and Proliferation of CT

c. 1975

c. 2011
Overview

Computed Tomography
Generations
Natural history and new technology

3D Image Reconstruction
Filtered backprojection
Basics and practicalities
Open-source resources

Image Quality
Artifacts
Spatial Resolution
Contrast Resolution
Noise

Proliferation and Applications
Diagnostic imaging
Image-guided interventions
First-Generation CT

**Scan and Rotate:**
Linear scan of source and detector
Line integral measured at each position: $p(\xi)$

$$p(\xi; \theta) = p_0 e^{-\int_0^{SDD} \mu(x,y,z) dy}$$

Rotate source-detector $\Delta \theta$
Repeat linear scan…
Projection data: $p(\xi, \theta)$
CT “Generations”

1\textsuperscript{st} Generation (1970)

Pencil Beam Translation / Rotation

2\textsuperscript{nd} Generation (1972)

Fan Beam Translation / Rotation
CT “Generations”

3rd Generation (1976)
Fan Beam
Continuous Rotation

4th Generation (1978)
Fan Beam
Continuous Tube Rotation
Stationary Detector

Helical (Spiral CT)

Pitch = \frac{\text{Table increment / rotation (mm)}}{\text{Beam collimation width (mm)}}

**Pitch < 1:**
- Overlap
- Higher z-resolution
- Higher dose

**Pitch > 1:**
- Non-overlap
- Lower z-resolution
- Lower patient dose
Dual-Source CT

Two complete x-ray and data acquisition systems on one gantry.
330 ms rotation time
(effective 83 ms scan time)

Siemens Healthcare– Somatom Definition
Recent Advances: *Cone-Beam CT*

**Fully 3-D Volumetric CT**

**Conventional CT:**
- Fan Beam
- 1-D Detector Rows
- Slice Reconstruction
- Multiple Rotations

**Cone-Beam CT:**
- Cone-Beam Collimation
- Large-Area Detector
- 3-D Volume Images
- Single Rotation
Filtered Backprojection: The Basics
Implementation

1. Projection at angle $\theta$: $p(\xi, \theta)$
2. Filtered Projection: $g(\xi, \theta)$
3. Backproject $g(\xi, \theta)$ and add to image $\mu(x,y)$

Loop over all views (all $\theta$)
The Sinogram $p(\xi, \theta)$

**The Sinogram:**
Line integral projection $p(\xi)$
... measured at each angle $\theta$
$\rightarrow p(\xi; \theta)$ “Sinogram”

\[ p(\xi; \theta) \]
**Simple Backprojection:**

Trace projection data $p(\xi;\theta)$ through the reconstruction matrix from the detector ($\xi$) to the source.

Simple backprojection yields radial density $(1/r)$.

Therefore, a point-object is reconstructed as $(1/r)$.

**Solution:** “Filter” the projection data by a “ramp filter” $|r|$.
The Filtered Sinogram:
Convolve with RampKernel(\(\xi\))
\(p(\xi) \ast \text{RampKernel}(\xi)\)
Equivalent to Fourier product
\(M(f_x)|f_x|\)

Filtered Backprojection
Filtered Backprojection

Projection Data \( p(\xi, \theta) \)

3D Reconstruction (Axial Slice) \( \mu(x,y,z) \)
Cone-Beam Geometry
Definitions and Coordinate Systems
3D Filtered Backprojection

Raw Projection Data

\[ p(u,v;\theta) = p_0 e^{-\int_0^{SDD} \mu(x,y,z) dy} \]
Offset-Gain (and Defect) Correction

\[ I_{\text{proc}}(u, v) = K \frac{I_{\text{raw}}(u, v) - I_{\text{offset}}(u, v)}{I_{\text{gain}}(u, v) - I_{\text{offset}}(u, v)} \]
Log Normalization

\[ p_1(u, v; \theta) = \ln \left( \frac{p_0}{p(u, v; \theta)} \right) \]

\[ SDD = \int_0^{SDD} \mu(x, y, z)dy \]
3D Filtered Backprojection

Cosine Weighting (Feldkamp Weights)

\[ p_2(u, v; \theta) = p_1(u, v; \theta) \left[ \frac{SDD}{\sqrt{SDD^2 + u^2 + v^2}} \right] \]
Data Redundancy (Parker) Weights

\[ p_3(u, v; \theta) = p_2(u, v; \theta)w_3(u; \theta) \]

\[
w_3(u; \theta) = \begin{cases} 
\sin^2 \left( \frac{\pi \theta}{4 \left( \frac{1}{2} \phi_{fan} - \phi(u - u_0) \right)} \right) & \text{for } 0 \leq \theta \leq \phi_{fan} - 2\phi(u - u_0) \\
1 & \text{for } \phi_{fan} - 2\phi(u - u_0) \leq \theta \leq \pi - 2\phi(u - u_0) \\
\sin^2 \left( \frac{\pi (\pi + \phi_{fan} - \theta)}{4 \left( \frac{1}{2} \phi_{fan} + \phi(u - u_0) \right)} \right) & \text{for } \pi - 2\phi(u - u_0) \leq \theta \leq \pi + \phi_{fan} \end{cases}
\]
Ramp Filter

\[ p_4(u, v; \theta) = FT^{-1} \left[ FT \left[ p_3(u, v; \theta) \right] |\rho| \right] \]

\[ = p_3(u, v; \theta) \star \left( -\frac{1}{2\pi^2 u^2} \right) \]
3D Filtered Backprojection

Smoothing / Apodization Filter

\[ p_5(u, v; \theta) = FT^{-1}[FT[p_4(u, v; \theta)]T_{win}(f)] \]
\[ = p_4(u, v; \theta) * t_{win}(u, v) \]
\[ T_{win}(f) = h_{win} + (1 - h_{win}) \cos(2\pi f \Delta) \]
Evolution of the Projection Data
3D Filtered Backprojection

Repeat \times \sum \# \text{ of voxels} \# \text{ of projections}
Evolution of the 3D Recon
Open-Source Resources

OSCaR
Open-Source Cone-Beam Reconstructor

MATLAB
Source code (m)
Function call (m)
Executable UI (exe)

Intended user
Education
Research

Input data types
jpg, raw, png, etc.

Flexible, transparent
Filters, voxel size, Reconstruction filters

www.jhu.edu/istar/downloads
Open-Source Resources

PortoRECO
Portable Reconstruction

C# / C++
Source code (C#)
Executable UI (exe)

Intended user
Education
Research

Input data types
jpg, raw, png, etc.

Flexible, transparent
Filters, voxel size, Reconstruction filters

www.jhu.edu/istar/downloads
Artifacts
A big problem for cone-beam CT:
SPR is very large for large cone angles (i.e., large FOV)

1) Reduced contrast
   Reduction of $\Delta CT$
2) Image artifacts
   Cupping and streaks
3) Increased image noise
   Reduced DQE
   Reduced soft-tissue detectability

Artifacts: X-ray Scatter

$$C' = C_o + \frac{1}{\alpha d} \ln \left( \frac{1 + SPR \ e^{-C_o \alpha d}}{1 + SPR} \right)$$
Artifacts: X-ray Scatter

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A big problem for cone-beam CT:
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Image Quality

Key Image Quality Metrics
- Image Uniformity
- CT # Accuracy
- Spatial resolution
- Contrast resolution
- Noise (and NPS)
- CNR
- NEQ
Uniformity / Stationarity

- **Signal Uniformity**
  - Stationarity of the mean
  - Shading artifacts
  - Beam-hardening
  - Truncation

- **Noise Uniformity**
  - Stationarity of the noise
  - WSS of second-order statistic
    - **Physical effects:**
      - Quantum noise
      - Bowtie filter
    - **Sampling effects:**
      - Intrinsic to FBP
      - Number of projections
      - View aliasing

\[ \Delta_{HU} = (4.6 - 1.3) \text{ HU} = 3.3 \text{ HU} \]
Uniformity / Stationarity

• Signal Uniformity
  Stationarity of the mean
  Shading artifacts
  Beam-hardening
  Truncation

• Noise Uniformity
  Stationarity of the noise
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  Physical effects:
    Quantum noise
    Bowtie filter
  Sampling effects:
    Intrinsic to FBP
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    View aliasing

Variance Maps $\sigma^2(x,y)$

\(\sigma^2(/\text{mm})^2\)
Spatial Resolution (line-pairs per mm)

Minimum resolvable line-pair group
Spatial Resolution
(Modulation Transfer Function)
Spatial Resolution
(Modulation Transfer Function)

127 μm Wire in H₂O

\[ MTF(f_x, f_y) = |FT[LSF(x, y)]| \]
CT image noise depends on:
- Dose
- Detector efficiency
- Voxel size
  - Axial, $a_{xy}$
  - Slice thickness, $a_z$
- Reconstruction filter

Barrett, Gordon, and Hershel (1976)

\[
\sigma^2 = \frac{k_E}{D_0} \frac{1}{\eta a_{xy}^3 a_z} K_{xy}
\]

\[
\sigma \propto \sqrt{\frac{1}{D_0}} \propto \sqrt{\frac{1}{a_{xy}^3}} \propto \sqrt{\frac{1}{a_z}}
\]
Image Noise

Dose

\[ \sigma \sim a + \frac{b}{\sqrt{X}} \]

Reconstruction Filter

Noise (CT #) vs. Dose (mGy)

PSF Width (mm) vs. Reconstruction Window Parameter

Voxel Noise (mm⁻¹) vs. Reconstruction Window Parameter

Dose Reconstruction Filter
Noise-Power Spectrum

- The NPS describes:
  - Frequency content (correlation)
  - Magnitude of fluctuations

\[
NPS(f_x, f_y, f_z) = \frac{a_x}{L_x} \frac{a_y}{L_y} \frac{a_z}{L_z} \left\langle \left| \text{DFT} \{\Delta I(x, y, z)\} \right|^2 \right\rangle
\]

Magnitude of fluctuations

\[
\sigma^2 = \iiint NPS(f_x, f_y, f_z) df_x df_y df_z
\]
Noise-Power Spectrum

Axial Plane \((x, y)\)

Axial NPS

\[
S(f_x, f_y)
\]

Spatial Frequency, \(f_x\) (mm\(^{-1}\))

Spatial Frequency, \(f_y\) (mm\(^{-1}\))
Noise-Power Spectrum

Sagittal Plane \((x,z)\)

Sagittal NPS

\[ S(f_x, f_z) \]

Spatial Frequency, \(f_x\) (mm\(^{-1}\))

Spatial Frequency, \(f_z\) (mm\(^{-1}\))
Noise-Power Spectrum

\[ NPS(f_x, f_y, f_z) \]

- **Axial domain:**
  - "Filtered-ramp"
  - *Mid-Pass*

- **Longitudinal domain:**
  - "Band-limited"
  - *Low-Pass*
Contrast

A “large-area transfer characteristic”

Defined:

- As an **absolute** difference in mean pixel values:
  
  \[
  C = |\mu_1 - \mu_2|
  \]
  
  **For example:**
  
  \[
  C = |0.18 \text{ cm}^{-1} - 0.20 \text{ cm}^{-1}| = 0.02 \text{ cm}^{-2}
  \]
  
  or
  
  \[
  C = |-100 \text{ HU} - 0 \text{ HU}| = 100 \text{ HU}
  \]

- As a **relative** difference in mean pixel values:
  
  \[
  C = \frac{|\mu_1 - \mu_2|}{(\frac{\mu_1 + \mu_2}{2})/2}
  \]
  
  **For example:**
  
  \[
  C = \frac{|0.18 \text{ cm}^{-1} - 0.20 \text{ cm}^{-1}|}{0.19 \text{ cm}^{-1}} \approx 10\%
  \]
Contrast-to-Noise Ratio

Soft-Tissue-Simulating Spheres

Dose to Isocenter (mGy)

CNR

Contrast to Noise Ratio

Soft Tissue Simulating Spheres

100 kVp

103 HU

88 HU

66 HU

45 HU

25 HU

22 HU

11 HU

23.3 mGy

9.6 mGy

2.9 mGy

0.6 mGy
Proliferation of CT Technologies and Applications

Diagnostic Imaging

Specialty Applications and Image-Guided Procedures

Trade names removed.
Proliferation of CT Technologies and Applications

Breast Screening / Diagnosis

Iodine-Enhanced Tumor Imaging

J. M. Boone (UC Davis)
Proliferation of CT Technologies and Applications

Upper Extremities

Morphology, Function, and Quantitation

Dual-Energy CBCT

Lower Extremities

Weight-Bearing

Iodine
Calcium
Proliferation of CT Technologies and Applications

Skull Base Surgery

Spine / Orthopaedics

Thoracic Surgery

CBCT C-Arm
Proliferation of CT Technologies and Applications

Head and Neck

Sarcoma

Lung

Prostate
Proliferation of CT
Increased utilization (and related challenges)
New technology (e.g., cone-beam CT)
Specialty applications

Open Source
OSCaR, PortoRECO
Others? (Please contact me.)

More to Come – This Week:
1.) Siewerdsen – Filtered Backprojection Fundamentals (MON-A-311)
2.) Fessler – Iterative Image Reconstruction (TUE-A-211)
3.) Yu – Optimization of image acquisition and recon (WED-E-110)

Information and Handouts:
www.jhu.edu/istar