External Beam Photon Dose Calculations

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I am a founder and Chairman of TomoTherapy Inc. (Madison, WI) which is participating in the commercial development of helical tomotherapy.
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Correction-Based Dose Computation

• Based on measured dose distributions in a water phantom.

• Independent corrections for:
  – Beam modifiers
  – Surface contours
  – Tissue heterogeneities

• Based on effective spatial measures such as depth, field boundaries and path lengths.

• Heterogeneity corrections that produce less accuracy than having no corrections at all.
Model-Based Dose Computation

• The convolution/superposition, finite pencil beam and Monte Carlo methods are model-based dose calculation systems.

• Unlike correction-based methods, model-based methods compute the dose directly in a patient representation.

• Model-based dose computations use monitor unit calculations based on beam intensity (e.g., energy fluence) rather than dose in a phantom.

• The beam has to be modeled explicitly.
What Has to Be Accounted For In Model Based Methods

- Finite source size.
- Angular distribution of photons.
- Primary transmission.
- Extrafocal radiation mainly from the primary collimation and the field flattening filter.
- Differential hardening of the beam by the field flattening filter.
- Curved leaf ends.
- Leaf configuration.
- Tongue and groove effect.
- Leaf transmission.
- Electron contamination.
- Tissue heterogeneities.
Modeling the Head

Percent of Energy Fluence at Central Axis

Direct ~85-95%
Primary Collimator ~1-4 %
Field Flattening Filter ~5-10 %
Issues with the Leaf Shape

Potential errors due to leaf shaping

Rounded Edge

Tongue & Groove

From Bruce Curran
Model-Based Methods Are Need for IMRT

- IMRT = Summation of small fields
- Dose = function(\text{penumbra} + \text{leakage} + \text{head scatter})
- Need accurate treatment head model to get this right

From Michael Sharpe, U. of Toronto
IMRT Is About Using Small Fields

Intensity pattern possibly unconstrained intensity levels

Intensity limited to a few discrete intensity levels

What is delivered

Adapted from Michael Sharpe, U. of Toronto
IMRT Is About Using Small Fields

- Accuracy of dose model at small field sizes is a consideration
- Convolution-superposition or Monte Carlo desirable

From Michael Sharpe, U. of Toronto
Extend the Beam Model

- Minimum Equivalent Square
  - Conventional XRT: ~4x4cm²
  - IMRT: ~0.5x1cm² - 2x2cm²

- How to get small field data?
  - Direct measurement – difficult, errors
  - Extrapolation of existing data – assumes an underlying physical model
  - Verify a “golden model”

- Any case will have uncertainty; evaluate its impact on a clinical IMRT distribution generated by the inverse planning system
Reasons for Drop in Output with Small Field Size

- Backscatter into monitor unit from beam defining jaws
- Reduced scatter (phantom and head)
- Electronic disequilibrium
- Obscuration of the source
Dose Computation as a Linear System

In particular:

\[ D_1 = M_{11} W_1 + M_{12} W_2 \]
\[ D_2 = M_{21} W_1 + M_{22} W_2 \]

In general:

\[ D_i = \sum_j M_{ij} W_j \quad \text{or} \quad D = MW \]

Where M is a matrix giving the contribution of dose from each pencil beam.
Pencil Beam Convolution

\[ D(x, y, z) = \iiint \Psi(x', y', z) K_z(x - x', y - y') \, dx \, dy \]

From Colin Field, University of Alberta
Pencil Beams Generated for Homogeneous Water are Fast

- Fast especially for iterative optimization calculations.
- The importance of speed grows for more beam directions, finer resolution, and more iterations.
- Good for early phases of iteration.
- Should do a more accurate calculation for the last iteration and final dose calculation.
The Problem with Pre-Computed Pencil Beams

The scattering of rays near the boundary are computed with an environment of water around it when it, in fact, may not always exist.
The dose distribution can be computed using a convolution equation:

\[ D(\vec{r}) = \int_{\text{vol}} \frac{\mu}{\rho} \Psi(\vec{r}') A(\vec{r} - \vec{r}') \, dV \]

- **Dose Distribution**
- **Primary Energy Fluence**
- **Kernel Derived From Monte Carlo Simulation**
- **Source of Radiation**
- **Primary Interaction Site**
- **Dose Deposition Site**
Graphical Representation of Dose Computation using Convolution

Primary Energy Fluence Distribution

Convolution Kernel (from MC Simulation)

Dose Distribution

Adapted From Colin Field
Comparison of Two Kernel Sets

Mackie et al. 1988
Chui and Mohan 1986

Mackie et al. 1988 PMB 33:1-20
Spherical Shell Summed Kernels

Mackie et al 1988 PMB 33:1-20

Diagram 1: Fraction of energy deposited within shell as a function of radius for different types of scattering.

Diagram 2: Fraction of energy deposited within shell as a function of radius for different types of scattering, including bremsstrahlung and annihilation.

Legend:
- Total
- Primary
- First scatter
- Second scatter
- Multiple scatter
- Bremsstrahlung plus annihilation

Notes:
- "Name of the kernel category. There is no annihilation dose at this energy"
Kernels Describe Photon Scattering and Electron Transport

10 cm x 10 cm Parallel Beam In Water

Ahnesjo and Asparadakis, 1999 Phys Med Biol 44:R99-R155
Adjusting the Spectra to Get Agreement with Measurements

The depth-dose curve is very forgiving with respect to spectral shape.
Source of Photons for a Conventional Linac

Scatter Fraction from the Flattening Filter

Ahnesjo 1994 Med. Phys. 21;1227-1235
MC Simulation of the Relative Fluence as a Function of Radius in the Beam

MC Simulation of the Radial Dependence of Mean Energy

10 MV Varian

MC Simulation of the Radial Dependence of Mean Energy

6 MV Elekta

Tomotherapy has little off-axis spectral dependence because it has no flattening filter; favorable for TPS

Modeling the Energy Fluence from Measurements in Air

Comparison of an energy fluence model and the dose in air. Used to model energy fluence for MC simulation but also could be used for the convolution calculations.

Effect of Kernel Hardening

Depth-Dose at 85 cm SSD for 1 cm Beam Width

Measurement

Pinnacle Model

Dose

Depth (cm)
Percent Depth-Dose (Normalized to Dose at 10 cm Depth)

Tomotherapy Beam

Error bars 2% and 1 mm

Field size 5 x 40 cm²
SSD = 85 cm
Transverse Unmodulated Dose Profiles

Calculations using the convolution/superposition method on the TomoTherapy HI-ART II treatment planning system.
Longitudinal (Z-axis) Dose Profiles

Tomotherapy Beam

Z profiles on depth at x = 0
Field size 5 x 40 cm²
SSD = 85 cm
Superposition Equation

\[ D(\vec{r}) = \int_{\text{vol}} Z_T(\vec{\rho}_{\vec{r}'} \cdot \vec{r}') A(\vec{\rho}_{\vec{r}-\vec{r}'} \cdot \vec{r} - \vec{r}') \, dV \]

- The Primary Term Depends on the Radiological Pathlength of the Primary Ray
- The Kernel is Scaled w.r.t. the Radiological Pathlength Between the Primary Photon Interaction Site and the Dose Deposition Site
Superposition Assumptions

- Ray tracing from the primary photon interaction site to the dose deposition site assumes that that is the relevant path.
- This correctly accounts for perturbation in first scatter photons which is the dominant scatter.
- It is not a bad approximation for electrons.
- The approximation is worse for 2nd and higher order scatter.
No Kernel Stretching

muscle $\rho = 1 \text{ gr/cm}^3$

lung $\rho = 0.25 \text{ gr/cm}^3$

Image from Nikos Papanikolaou, Ph.D.
Heterogeneities: Kernel Stretching

muscle $\rho = 1 \text{ gr/cm}^3$

lung $\rho = 0.25 \text{ gr/cm}^3$

Image from Nikos Papanikolaou, Ph.D.
Homogeneous Vs. Heterogeneous

Heterogeneous Terma But Homogeneous Electron Transport and Scatter

Homogeneous Terma, Electron Transport and Scatter
Non-Invariance of Convolution Kernels in Inhomogeneous Phantoms

From Woo and Cunningham
Superposition
Result

In low density materials like lung (or cork) the electrons and scattered photons travel much farther.

To first order, radiation penetration is inversely proportional to electron density.

From Anders Ahnesjo
Small Field Sizes and Electronic Disequilibrium

6 MV Direct Component Only

Each Beam Normalized Separately to Dmax at the Central Axis
Dose per Incident Energy Fluence
As a Function of Field Diameter

A=Adipose, M=Muscle, B=Bone, L=Lung  4 MV, Parallel Beam

Ahnesjo and Asparadakis, 1999 Phys Med Biol 44:R99-R155
Sweeping Contamination Electrons With an Electromagnet

Electrons Swept Out of the Page and Positrons Swept Into the Page

From Jursinic and Mackie PMB 41:1499-1509
Buildup Curves With and Without Magnetic Field

From Jursinic and Mackie PMB 41:1499-1509
Surface Dose as a Function of Magnetic Field Strength

Each Beam Normalized Separately to the Dmax at the Central Axis with No Magnetic Field

From Jursinic and Mackie PMB 41:1499-1509
Ionization Attributable to Electron Contamination

Electron Contamination Depth-Dose Curve Has a Small Buildup at Higher Energies

From Jursinic and Mackie PMB 41:1499-1509
Electron Contamination

**Electron Contamination Dose Modeling**

\[ \text{E.C. Dose}(f_s, r, d) = F_{\text{Depth}}(d, f_s) \cdot F_{OA}(r) \]

- \( f_s \) = Field Size
- \( r \) = Off Axis Distance
- \( d \) = Depth

**Off Axis Effect**

\[ F_{OA}(r) = e^{-OAC \cdot r^2} \]

**Field Size Effect**

\[ F_{FS}(f_s) = ECD_{10 \times 10} + C_1 \cdot (f_s - 10) \]
\[ + C_2 \left( e^{-C_3 \cdot 10} - e^{-C_3 \cdot f_s} \right) \]
Done as part of the modeling process – e contamination dose is modeled as a modified exponential curve.
Convolution Superposition as Done in Pinnacle

1. Model the incident fluence as it exits linac head.
2. Project this fluence through the density representation of the patient (CT data) – compute TERMA.
3. 3D convolution/superposition of TERMA and point kernel (ray tracing)
4. Electron contamination added to dose distribution at the end.
Monte Carlo Calculation

- Explicitly transport individual photons through the head and patient.
- Geometry and radiation physics is the model.
- EGS4 and BEAM, MCNP, Geant, and Penelope are available for research.
- Models are clinically available (e.g. Monaco from CMS)
Head and Neck Case

Histogram of Monte Carlo - Superposition

From Jeff Siebers
Lung Case

Histogram of Monte Carlo - Superposition

From Jeff Siebers
Comparisons for an IMRT Case

Monte Carlo

Convolution/superposition

Pencil Beam

From Robert Jeraj, University of Wisconsin
## Errors in Optimized Planning

### Convolution/superposition dose calculation

<table>
<thead>
<tr>
<th>Error [% $D_{\text{max}}$]</th>
<th>Tumour</th>
<th>Lung</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic</td>
<td>$-0.1 \pm 2$</td>
<td>$-1 \pm 1$</td>
</tr>
<tr>
<td>Convergence</td>
<td>2-5</td>
<td>1-4</td>
</tr>
</tbody>
</table>

### Pencil beam dose calculation

<table>
<thead>
<tr>
<th>Error [% $D_{\text{max}}$]</th>
<th>Tumour</th>
<th>Lung</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic</td>
<td>$+8 \pm 3$</td>
<td>$+6 \pm 5$</td>
</tr>
<tr>
<td>Convergence</td>
<td>3-5</td>
<td>6-7</td>
</tr>
</tbody>
</table>

Convolution/superposition dose calculation is acceptable, pencil beam dose calculation may not always be.
MU Calculations For Model Based Systems

\[
\text{MU} = \frac{D_{\text{presc}}(P)}{D / \text{MU}(P)}
\]

\[
\frac{D}{\text{MU}}(P) = \frac{D}{\Psi}(P) \frac{\Psi}{\text{MU}}(\text{SPD}, r)
\]
Beam Output for Model-Based Systems

- Establish Energy Fluence/MU By Measurement in a Phantom.
- But the Result is Phantom Independent.

\[
\frac{\Psi}{MU}(SPD, r) = \frac{D}{MU}(Ref) / \frac{D}{\Psi}(Ref)
\]

Beam Output Which is Phantom Independent

Measured

Computed for Same Condition as Measurement
Extended Phantom Concept and Dose Reconstruction
Will Monte Carlo Take Over?

- For few-field IMRT Monte Carlo is slower than the convolution/superposition method.
- The computation time for Monte Carlo is nearly independent of the number of beam directions.
- The Monte Carlo method is only slightly more accurate for photon beams.
- It will be easier to take into account IMRT delivery geometries (e.g. collimators).
Summary

• The convolution method is a deterministic calculation using results of Monte Carlo simulation.

• Beam characterization can be performed using Monte Carlo simulation.

• The superposition method, a variant of the convolution method, and predicts dose within a few percent of the Monte Carlo method and about an order of magnitude faster for few calculation fields.

• Energy fluence calibration is more appropriate for model-based dose calculation systems.